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Accelerating the Mitigation of Greenhouse Gas Emissions: The Influence of Uncertainties in Economic Growth and Technological Change

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Abstract

Technological change has been used both in favor of, and against delaying the onset of greenhouse gas abatement. In this paper we develop a theoretical model using stochastic dynamic programming to show that the uncertainties in technological change and economic growth have a direct impact on the design of cost-effective policies and their effect is to unambiguously *dilute* arguments in favor of delaying mitigation. Optimal strategies that that meet emission reduction targets in the presence of these uncertainties require earlier and greater abatement of carbon emissions. Policies aimed at meeting GHG reduction targets should recognize the benefits of early abatement.

Keywords: Timing of Mitigation; Climate Change; Technological Change; Economic Uncertainty

1 Introduction

During the Kyoto period (2005–2012) nations have taken very different approaches to mitigation of Greenhouse Gases (GHGs). The US has avoided taking any steps at the national level to reduce GHGs and chosen to focus instead on developing new technologies for abatement in the future; EU member states have taken a more proactive approach to GHG mitigation and some have made progress towards meeting their Kyoto targets. Others such as Canada have made feeble attempts to reduce carbon emissions, implicitly choosing to delay the onset of mitigation. As nations look to the Post-Kyoto period starting

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2012, questions of delaying or accelerating emissions reductions will once again come to the fore in international discussions.

An issue that has been the center of a long debate, and is also the subject of this work, relates to the timing of mitigation decisions (Grubb et al., 1995). Scholars have argued that early abatement is required in order to encourage the development of energy efficient technologies (Goulder & Schneider, 1997); to militate against the effects of inertia in energy systems (Azar & Dowlatabadi, 1999); to counter the tendency of political systems to procrastinate (Kelbekken & RIve, 2007). Others have argued that a delay would reduce costs: through exogenous technological change and returns to investments in R&D; by taking advantage of capital turnover; and through discounting and positive marginal productivity of capital (Wigley et al., 1996; Manne & Richels, 1999; Morgenstern et al., 1999).

More recently, scholars have also debated the timing of carbon reductions vis-à-vis avoidance of the impacts of climate change. Azar & Dowlatabadi (1999) makes the obvious point that questions of timing merely depend upon the CO_2 concentration target chosen; a delay would have a lower impact on our ability to meet high target (e.g. 650 ppmv by 2100) than a low one (450 ppmv by 2100). O'Neill & Oppenheimer (2002) argue that CO_2 concentrations above 450 ppmv constitute "dangerous" climate change and make the case that a delay in mitigation beyond 2010 would make it impossible to meet a 450 ppmv target. Yohe et al. (2004) show that modest emissions reductions in the near-term serve as a hedge against high values of climate sensitivity and that uncertainty "is reason for acting in the near term".

While considerations of climate outcomes such as global temperature are important in setting global emissions reductions goals (Hasselmann et al., 2003), Kyoto-style targets and timetables continue to be the preferred approach among policymakers[1]. This paper focuses on the timing of emissions within the context of emissions reduction targets relative to a current emissions baseline. An important issue that has received less attention in the timing debate is the role of uncertainties in technological change and future economic growth. These uncertainties can arise for a number of different reasons including ups and downs in economic growth, fluctuations in energy prices and prices of commodities, and uncertainties in the fraction of energy supply that will be met by different fuel types. Since assumptions about technological change and economic growth are central to future emissions and abatement scenarios, it is likely that uncertainties in these quantities could impact the magnitude and timing of emissions reductions. Specifically, the question is whether taking account of these uncertainties provides additional reasons to accelerate emissions reduction, or whether this would suggest delaying reductions? In this paper we show that these unambiguously dilute arguments that favor delaying mitigation. In other words, the key result of this paper is that meeting emission reduction targets in the presence of these uncertainties results in optimal strategies that require

[1]Witness the recent legislation on climate change in California which defines targets of 1990 levels in 2020, and 80% below 1990 levels in 2050; or British Columbia legislation which sets a target of 20% below 1990 levels in 2020.

earlier and greater abatement.

2 Model Framework

We use a dynamic programming framework. Since much of the discussion of mitigation is centered on strategies that minimize the mitigation cost, dynamic programming is a useful way to characterize the problem. The problem is to find a mitigation path to the target emission $E_{GHG}(T)$, which minimizes the total discounted cost:

$$TC(T) = \int_0^T Cost(t)e^{\rho t}dt \tag{1}$$

GHG emissions are a byproduct of economic activity and directly proportional to the total economic output $Q(t)$. $\sigma_t$is the "decarbonization" function or the carbon intensity, i.e., the ratio of carbon emissions to the total economic output. This implies that the emissions (at time t), in the absence of abatement policies, is given by $E_0(t) = \sigma(t)Q(t)$.

The abatement *level* is given by: $\lambda(t) = 1 - \frac{E_\lambda(t)}{E_0(t)}$, where $E_\lambda(t)$ is GHG emission level in the presence of abatement policies, $\lambda(t)$ appears in the relation between and the total economic output $Q(t)$:

$$E_\lambda(t) = \Big(1 - \lambda(t)\Big)\sigma(t)Q(t) \tag{2}$$

Technological change is conceptually divided into two categories—exogenous and endogenous. Exogenous technical change causes changes to the emissions to GDP ratio in the absence of abatement policies, and $\sigma(t)$ is independent of $\lambda(t)$. Endogenous technological change results from abatement policies; here $\sigma(t)$ is a function of $\lambda(t)$. Endogenous technological change is a strong argument in favor of engaging in early mitigation (Goulder & Schneider, 1997; Grubler et al., 1999). Since, as we show in this paper, the inclusion of uncertainties in economic and (exogenous) technological change is consistent with this finding, we limit our analysis to exogenous technical change. Further, the influence of uncertainties in economic growth alone on timing of abatement provides a rationale for this analysis independent of the relationship between $\sigma(t)$ and $\lambda(t)$. Ignoring endogenous technical change greatly simplifies the mathematics, and does not weaken our conclusions on the role of technical and economic uncertainty in meeting targets. We will return to the issue of endogenous technical change in the final section of this paper.

The approach assumes that mitigation policy is a dynamic process. The goal of mitigation is to ensure that the mitigated emission level $E_\lambda(t)$ stays on the optimal trajectory, such that the emissions at a future time T meet a predetermined target $E_{GHG}(T)$ while minimizing the cost given in Equation 1.

To achieve this goal, the changes in GHG emission ΔE_λ are computed in reaction to the changes ΔE_0 in the value of $\sigma(t)Q(t)$, the base emissions.

In this formulation, dynamic constraint of the problem is given by:

$$\dot{E}_\lambda = \Big(1-\mu(t)\Big)\sigma(t)Q(t)\left[\frac{\dot{\sigma}(t)}{\sigma(t)}+\frac{\dot{Q}(t)}{Q(t)}\right] \tag{3}$$

The control variable, $\mu(t)$, is given by:

$$\mu(t) = 1-\frac{\dot{E}_\lambda}{\dot{E}_0} \tag{4}$$

The relation between the "dynamical abatement level" $\mu(t)$ and the abatement level $\lambda(t)$ is found by differentiating Equation 1 with respect to time:

$$\mu(t) = \lambda(t)+\frac{\dot{\lambda}(t)}{\frac{1}{\sigma(t)Q(t)}\cdot\frac{\delta\sigma(t)Q(t)}{\delta t}} = \lambda(t)+\frac{\dot{\lambda}(t)}{\left[\frac{\dot{\sigma}(t)}{\sigma(t)}+\frac{\dot{Q}(t)}{Q(t)}\right]} \tag{5}$$

Whereas the abatement level is constrained to satisfy $0 \leq \lambda(t) \leq 1$ (Equation 2), the control variable $\mu(t)$, can become larger than one. If $\mu(t) > 1$, it follows that, $\dot{E}_\lambda(t) < 0$. To maintain the GHG emission stationary requires $\mu(t) = 1$. Further, at the onset of mitigation, $\lambda(0) = 0$ for all the mitigation effort, and therefore the mitigation cost is in the rate of implementation $\dot{\lambda}(t)$.

Without loss of generality for our conclusions, we can assume that $\sigma(t)$ and economic output $Q(t)$ on the average follow a simple exponential evolution, i.e. $\sigma(t) \approx \sigma_0 e^{\delta_\sigma t}$ and $Q(t) \approx Q_0 e^{\delta_Q t}$, so that $\frac{\dot{\sigma}_i(t)}{\sigma_i(t)} = \delta_\sigma \quad (\delta_\sigma < 0)$ and $\frac{\dot{Q}(t)}{Q(t)} = \delta_Q$. This is the expression for $\sigma(t)$ and $Q(h)$ that we will use in the case of perfect foresight, i.e., when technological and economic uncertainties are not present.

2.1 Incorporating the uncertainties

Uncertainties in the "decarbonization" function and economic output imply that $\sigma(t)$ and $Q(T)$ can be treated as stochastic variables. The simple exponential form for their evolution is replaced by a set of stochastic differential equations (Dixit & Pindyck, 1993):

$$d\sigma(t) = \delta_\sigma\sigma\cdot dt+\eta_\sigma(\sigma)\cdot dz_\sigma \tag{6}$$

$$dQ(t) = \delta_Q Q\cdot dt+\eta_Q(Q)\cdot dz_Q \tag{7}$$

dz_{σ_i} and dz_Q are infinitesimal stochastic variables such that: $\langle dz_Q\rangle = \langle dz_\sigma\rangle = 0$ and $\langle dz_Q^2\rangle = \langle dz_\sigma^2\rangle = dt$, where $\langle x\rangle$ represents the expected value of x. Furthermore, we assume that they are uncorrelated, i.e: $\langle dz_Q\cdot dz_\sigma\rangle = 0\dot{\text{A}}$t the zero-stochasticity limit, (i.e. when the infinitesimal variances $\eta_\sigma(\sigma)$ and $\eta_Q(Q)\to 0$), the solutions to Equation 6 and Equation 7 are the simple exponentials $\sigma(t) \approx \sigma_0 e^{\delta_\sigma t}$ and $Q(t) \approx Q_0 e^{\delta_Q t}$.

Here we employ the common assumption that the infinitesimal variances $\eta_\sigma(\sigma)$ and $\eta_Q(Q)$ take the form $\eta_\sigma(\sigma) = \eta_\sigma \cdot \sigma$. This is equivalent to assuming that the stochasticity generates fluctuations proportional to the value of the variables, and that $Q(t)$ and $\sigma(t)$ are lognormally distributed random variables.

When $Q(t)$ and $\sigma(t)$ follow stochastic evolution given by Equation 6 and Equation 7, the dynamical constraint equation (Equation 4) becomes stochastic as well:

$$\begin{aligned} dE(t) &= \xi \cdot dt + \eta_E^\sigma \cdot dz_\sigma + \eta_E^Q \cdot dz_Q \\ \text{with} \qquad \xi &= \Big(1-\mu(t)\Big)\sigma(t)Q(t)\Big[\delta_\sigma + \delta_Q\Big] \end{aligned} \tag{8}$$

In summary the dynamic programming problem is to minimize Equation 1, subject to the dynamical constraints of Equation 3 (non-stochastic), or Equation 8 (stochastic) and with a condition on the expected final value of emissions given by $\langle E(T)\rangle = E_{GHG}(T)$.

3 Effect of stochasticity on meeting the target values

Before we proceed to solve the formulation described in Section 2, it is instructive to look the impact of stochasticity on the final "target" values for the emissions abatement level at time T. When $\sigma(t)$ and $Q(t)$ are stochastic, neither $\sigma(t)$ nor $Q(t)$ can be precisely known. In this section we estimate the consequence for the target abatement levels without stochasticity, $\lambda(T) = 1 - \frac{E_{GHG}(T)}{\sigma(T)Q(T)}$, and with $\lambda_{st}(T) = 1 - \left\langle \frac{E_{GHG}(T)}{\sigma(T)Q(T)} \right\rangle$.

Using Ito's lemma (Dixit & Pindyck, 1993) and Equation 8, one can write the stochastic differential equation for $\sigma(T)Q(N)$.

$$d\Big[\sigma(t)Q(t)\Big] = \sigma(t)Q(t)\Big(\big[\delta_\sigma + \delta_Q\big]dt + \eta_\sigma dz_\sigma + \eta_Q \cdot dz_Q\Big) \tag{9}$$

Rewriting Equation 9 by substituting $x = \ln(\sigma Q)$ gives:

$$dx = \big[\delta_\sigma + \delta_Q\big]dt + \eta_\sigma \cdot dz_\sigma + \eta_Q \cdot dz_Q \tag{10}$$

This corresponds to the equation for Brownian motion with respect of the variable x, where $\sigma(T)Q(T)$ is lognormally distributed with a time dependent variance. The distribution of values of $\sigma(T)Q(T)$ is given by $\phi(\sigma Q, T; \sigma_0 Q_0)$ which is the solution to the Kolmogorov equation (Dixit & Pindyck, 1993):

$$\frac{\delta\phi(x,t)}{\delta t} = \frac{1}{2}\Big(\eta_\sigma^2 + \eta_Q^2\Big)\frac{\delta^2\big[\phi(x,t)\big]}{\delta x^2} - \Big(\delta_\sigma + \delta_Q\Big)\frac{\delta\big[\phi(x,t)\big]}{\delta x} \tag{11}$$

The solution is (Karlin & Taylor, 1981):

$$\phi(\sigma Q, T; \sigma_0 Q_0) = \frac{1}{\sqrt{\pi T\left(\eta_\sigma^2 + \eta_Q^2\right)}} \cdot e^{\left[-\frac{\log\left(\sigma Q \big/ \sigma_0 Q_0 e^{[\delta_\sigma + \delta_Q]T}\right)^2}{\left(\eta_\sigma^2 + \eta_Q^2\right)T}\right]} \tag{12}$$

Equation 12 implies that

$$\lambda_{st}(T) = \langle \lambda(T) \rangle = 1 - \frac{E_{GHG}(T)}{\sigma_0 Q_0 e^{[\delta_\sigma + \delta_Q]T}} = \lambda(T) \tag{13}$$

The message of Equation 13 is that the addition of the stochastic effects does not affect the target abatement level $\lambda(T)$, although it introduces an uncertainty in its value. $\lambda(T)$ is lognormally distributed with a variance equal to $T\left(\eta_\sigma^2 + \eta_Q^2\right)$. The expected final value of emissions is given by:

$$\langle E(T) \rangle = E_{GHG}(T) \tag{14}$$

In other words, the target value can be met only in an expected sense and never exactly. Further, the variance of the emissions at the target increases with the time horizon T, and the level of stochasticity $\left(\eta_\sigma^2 + \eta_Q^2\right)$.

4 The Cost Function

A key ingredient of this normative framework is the cost function, whose structure determines the form of the solution. While the estimates of mitigation cost vary a lot, a number of these estimates can be expressed in the following functional forms (Nordhaus, 1994)

$$Cost(\lambda) = a\lambda^\nu \qquad a > 1 \tag{15}$$

The mitigation cost for carbon is a monotonic function of abatement and grows faster than linearly in the abatement level. This form of the cost function is consistent with reviews of top-down mitigation costs (Jaccard & Montgomery, 1996; Reilly et al., 2003) and the different values for the exponent are provided in Figure 1.

There has been a long debate between "top-down" and "bottom-up" camps on the cost of GHG mitigation. The top-down paradigm assumes that economies use the optimal mix of energy supply options given currently available technology. In the bottom up approach, the costs of carbon mitigation are calculated by replacing the current mix of technologies with more efficient technologies that are currently available or reasonably foreseen. In bottom-up approaches the energy savings from the use of efficient technologies result in abatement that has lower costs. The cost function $Cost(\lambda) = a\lambda^\nu$ with a and ν both greater

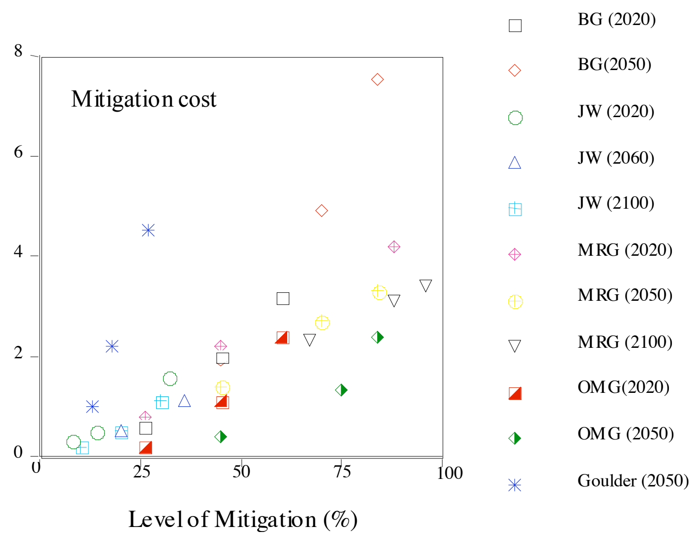

Figure 1: Mitigation cost (in percentage of GNP) as a function of mitigation level (in % of global carbon emissions) from the literature. Reproduced from Jaccard & Montgomery (1996). The cost function of each group can be approximated by the expression: $Cost(\lambda) = a\lambda^{\nu}$, but with different values for the parameters. Estimated values for exponent ν were estimated using a simple regression on the log of the cost model. Estimated values were 1.35(MRG), 1.223(JW), 2.027 (BG), 2.8 (OMG), 2.046 (Goulder).

than 1 better fits the "top-down" philosophy. The condition $\nu < 1$ offers some flexibility for incorporating "bottom up" arguments where-by costs of mitigation can be lowered by fast uptake of energy efficient technologies. In this work, however, we use $\nu > 1$, which is a more generally accepted assumption (see Figure 1). Some of the arguments in this paper would be reversed if we assume that $\nu < 1$.

We assume that the cost function is of a similar form but is a function of variable $\mu(t)$, i.e. we choose as cost function:

$$Cost(\mu) = a\mu^{\nu} \tag{16}$$

This second expression (Equation 16) involves both the level of abatement $\lambda(t)$, and rate of implementation $\dot{\lambda}(t)$ (Equation 5). This cost function $Cost(\mu) = a\mu^{\nu}$ allows us to incorporate an element akin to capital cost inertia in the formulation (Ha-Duong et al., 1997).

5 The Optimal Solution with Perfect Foresight

We first solve the problem defined by Equation 1 and Equation 2, with the cost function given by Equation 16, using dynamic programming. The objective function is:

$$I(t,T,\lambda) = \int_t^T \Pi_d dt = \int_t^T e^{-\rho t} Cost(\lambda) dt \tag{17}$$

The Bellman equation (Merton, 1964) for this problem is:

$$0 = \min_{\{\mu(t)\}} \left\{ \Pi_d(\mu,t) + \frac{\delta I_t}{\delta t} + \xi \frac{\delta I_t}{\delta E_\mu} \right\} \tag{18}$$

where ξ is given by Equation 8. The minimum occurs when the derivative of Equation 18 with respect to $\mu(t)$ vanishes. This implies that on the optimal path:

$$\frac{1}{\sigma(t)Q(t)\left[\delta_\sigma + \delta_Q\right]} \cdot \frac{\delta \Pi_d(\mu,t)}{\delta_\mu} = \frac{\delta I - t}{\delta E} \tag{19}$$

In order to transform Equation 18 into an equation for the optimal mitigation path, we derive the Bellman equation (Equation 17) with respect to the dynamical variable E to get:

$$\frac{1}{dt}\left\langle d\left[\frac{\delta I_t}{\delta E}\right]\right\rangle = 0 \tag{20}$$

Comparing Equation 20 and Equation 19, optimal path is given by:

$$\frac{d}{dt}\left[\frac{1}{\sigma(t)Q(t)}\cdot\frac{\delta\left[e^{-\rho t}Cost(\mu)\right]}{\delta\mu}\right]=0 \tag{21}$$

the functions $Cost(\mu) = a\mu^{\nu}$, $\sigma(t) = \sigma_0 e^{\delta_\sigma t}$ and $Q(t) = Q_0 e^{\delta_Q t}$ and Equation 21 together imply that the optimal path for the control variable is the exponential form:

$$\mu(t) = \mu(0)e^{\frac{(\delta_\sigma+\delta_Q+\rho)}{(\nu-1)}\cdot t} \tag{22}$$

In order to determine $\mu(T)$ and $\mu(0)$ one needs to know the optimal emission profile $E_\lambda(t)$ and the optimal mitigation profile $\lambda(t)$. They can be obtained by integrating Equation 4 and Equation 5, respectively. We assume that the abatement level at time $t = 0$ is zero, i.e., $\lambda(0) = 0$. Then, the optimal emission profile is given by the expression:

(23)

$$E_\lambda(t) = \frac{e^{[\delta_\sigma+\delta_Q]t}E_\lambda(0)\cdot\left\{\rho+\left(\nu-(\nu-1)\mu(0)e^{\left\{\frac{(\delta_\sigma+\delta_Q+\rho)}{(\nu-1)}\right\}t}\right)\cdot\left(\delta_\sigma+\delta_Q\right)\right\}}{\left\{\rho+\left(\nu-(\nu-1)\mu(0)\right)\cdot\left(\delta_\sigma+\delta_Q\right)\right\}}$$

And the equation of the optimal mitigation path is:

(24)

$$\lambda(t) = \left\{\frac{\mu(0)\left[e^{\left\{\frac{(\delta_\sigma+\delta_Q+\rho)}{(\nu-1)}\right\}t}-e^{-\left[\delta_\sigma+\delta_Q\right]t}\right]}{\left[\frac{(\delta_\sigma+\delta_Q+\rho)}{(\nu-1)}+\left[\delta_\sigma+\delta_Q\right]\right]}+\left(-1+[\delta_\sigma+\delta_Q]\right)\cdot\frac{\left(1-e^{-[\delta_\sigma+\delta_Q]\cdot t}\right)}{\left[\delta_\sigma+\delta_Q\right]}\right\}$$

The shape of the curves corresponding to Equation 23 and Equation 24 is shown in Figure 2.

From Equation 23 and Equation 24, and assuming T is large, we can derive an approximation for the initial value of the dynamical abatement $\mu(0)$:

$$\mu(0) \approx \lambda(T)\left(\frac{\Lambda_0}{(\delta_\sigma+\delta_Q)}+1\right)\cdot e^{-\Lambda_0 T} \tag{25}$$

where $\Lambda_0 = \frac{(\delta_\sigma+\delta_Q+\rho)}{(\nu-1)}$

Equation 22, Equation 24, and Equation 25 show that optimal abatement follows an exponential path whose rate depends on the value of rates for economic growth, decarbonization and discounting.

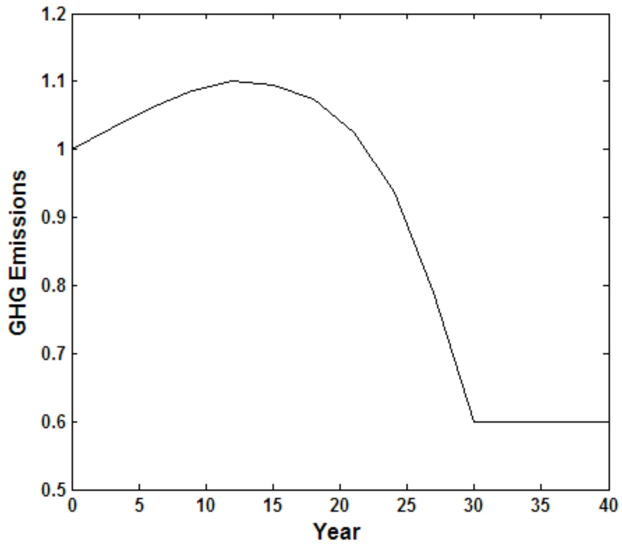

Figure 2: Typical form of an optimal emission profile $E_\lambda(T)$ for the non-stochastic assuming that after 30 years the emissions are stabilized to 40% below the value at $t = 0$. The unit of emission normalized to 1 at $t = 0$.

6 The Optimal Solution With Stochasticity

A key advantage of using dynamic programming is that it extends to the stochastic (Merton, 1971; Pindyck, 1980) case, i.e., where the dynamical constraint is given by Equation 8 instead of Equation 3. The expression to minimize is the expected total discounted cost $I(0, T, \lambda)$, where:

$$I(t, T, \lambda) = \Big\langle \int_t^T \Pi_d dt \Big\rangle = \Big\langle \int_t^T e^{-\rho t} Cost(\lambda) dt \Big\rangle \tag{26}$$

From Ito's Lemma, the Bellman equation becomes (Merton, 1971)

$$0 = \min_{\{\mu(t)\}} \Big\{ \Pi_d(\mu, t) + \frac{\delta I_t}{\delta t} + \xi \frac{\delta I_t}{\delta E_\mu} + \frac{1}{2} \Big\{ \big[(\eta_E^\sigma)^2 + (\eta_E^Q)^2 \big] \frac{\delta^2 I_t}{\delta E^2} \Big\} \Big\} \tag{27}$$

Following the same steps that lead to Equation 21, we get a stochastic generalization for the equation for the optimal trajectory:

$$\frac{1}{dt} \Big\langle d\Big[\frac{1}{\sigma(t) Q(t)} \cdot \frac{\delta \Pi_d(\mu, t)}{\delta \mu} \Big] \Big\rangle = \frac{1}{dt} \Big\langle d\Big[\frac{\delta I_t}{\delta E} \Big] \Big\rangle = 0 \tag{28}$$

Equation 28 is a bit more complicated to solve than Equation 21. We define:

$$\Psi(\sigma, Q, \mu, t) = \frac{\nu e^{-\rho t} a \mu^{\nu-1}}{\sigma(t)Q(t)} \tag{29}$$

Equation 28 reduces to $0 - \frac{1}{dt}\langle d[\Psi]\rangle$. Applying Ito's lemma yields the following equation for the optimal path of the control variable with stochasticity:

$$0 = \frac{1}{dt}\langle d[\Psi]\rangle = \left\{ -\rho\Psi + \alpha_\mu \frac{(\nu-1)}{\mu}\Psi - \Big[\delta_\sigma + \delta_Q\Big]\Psi + \Big[\frac{\eta_\sigma(\sigma)^2}{\sigma^2} + \frac{\eta_Q(Q)^2}{Q^2}\Big]\Psi \right\} \tag{30}$$

From Equation 30, (using $\alpha_\mu = \dot{\mu}$), the equation for the optimal path for the control variable $\mu(t)$ is :

$$\frac{d\mu}{dt} = \frac{\mu}{(\nu-1)}\Big\{\rho + \delta_Q + \delta_\sigma - \frac{\eta_\sigma(\sigma)^2}{\sigma^2} - \frac{\eta_\sigma(\sigma)^2}{Q^2}\Big\} \tag{31}$$

From Equation 31, the equation for the optimal path for the control variable $\mu(t)$ is:

$$\frac{d\mu}{dt} = \frac{\mu}{(\nu-1)}\Big\{\rho + \delta_Q + \delta_\sigma - \eta_\sigma^2 - \eta_Q^2\Big\} \tag{32}$$

Thus the stochastic version of the control variable is:

$$\mu_{st}(t) = \mu(0)e^{\frac{\left(\delta_\sigma + \delta_Q + \rho - (\eta_\sigma^2 + \eta_Q^2)\right)}{(\nu-1)}t} \tag{33}$$

At the zero stochasticity limit ($\eta_\sigma \to 0$ and $\eta_Q \to 0$), Equation 33 gives the same path for $\mu(t)$ as in Equation 22. Equation 32 implies that the effect of stochasticity is to reduce the average rate of change of $\mu(t)$, thus requiring a higher initial rate of abatement for meeting a given emission target $E_{GHG}(T)$.

Substituting $\mu_{st}(t)$ in Equation 5 and proceeding as in Equation 23 and Equation 24, the stochastic generalization of Equation 25 is obtained:

$$\mu_{st}(0) \approx \lambda_{st}(T) \cdot \left(\frac{\Lambda_\eta}{(\delta_\sigma + \delta_Q)} + 1\right)e^{-\Lambda_\eta T} \tag{34}$$

The impact on the onset of mitigation can be elicited from Equation 3 as:

$$\dot{\lambda}_{st}(0) = \Big(\delta_\sigma + \delta_Q\Big)\mu_{st}(0) \approx \lambda_{st}(T)\Big(\Lambda_\eta + (\delta_\sigma + \delta_Q)\Big)e^{-\Lambda_\eta T} \tag{35}$$

$$\text{with:} \qquad \Lambda_\eta = \Lambda_0 - \frac{(\eta_\sigma^2 + \eta_Q^2)}{(\nu-1)} = \frac{\left(\delta_\sigma + \delta_Q + \rho - (\eta_\sigma^2 + \eta_Q^2)\right)}{(\eta-1)} \tag{36}$$

Inspection of Equation 35 shows that $\dot{\lambda}_{st}(0) > \dot{\lambda}(0)$ when $\nu > 1$. This implies that the inclusion of stochasticity requires a faster *initial* rate of abatement in order to meet a given target. The size of the effect depends on the value of the target time T and of the rates of technological change and economic growth.

7 Quantitative Analysis of Results

In this section, we do a quantitative analysis of the influence of stochasticity on various aspects of the emissions abatement—total emissions reduction required to meet an emissions target, the cost of abatement, the time horizon of abatement policies, and questions related to delaying or accelerating mitigation.

The variables η_σ and η_Q control the uncertainty on the rate of change of the decarbonization function and on the growth rate of the economy. The assumption that $\eta_\sigma(\sigma) = \eta_Q \cdot \sigma$ and $\eta_Q(Q) = \eta_Q \cdot Q$, together with Equation 8, imply that $\sigma(t)$ and $Q(t)$ are lognormally distributed. If $\phi(\sigma, T; \sigma_0)$ and $\phi(Q, T; Q_0)$ denote the distributions of σ and Q, with $\sigma_0 = \sigma(t = 0)$ and $Q_0 = Q(t = 0)$ then:

$$\phi(\sigma, T; \sigma_0) = \frac{1}{\sqrt{2\pi t \eta_\sigma^2}} e^{\left[-\frac{\log\left(\frac{\sigma}{\sigma_0 e^{\delta_\sigma t}}\right)^2}{2\eta_\sigma^2 t}\right]} \tag{37}$$

$$\phi(Q, T; Q_0) = \frac{1}{\sqrt{2\pi t \eta_Q^2}} e^{\left[-\frac{\log\left(\frac{Q}{Q_0 e^{\delta_Q t}}\right)^2}{2\eta_Q^2 t}\right]} \tag{38}$$

Taking $t = 1$, in Equation 37 and Equation 38 yields a normal distribution for the annual rate of change δ_σ of $\delta(t)$ (there is a similar distribution for δ_Q):

$$\phi(\delta_\sigma, 1) = \frac{1}{\sqrt{2\pi \eta_\sigma^2}} e^{\left[-\frac{\left(\delta_\sigma - \delta_\sigma^{mean}\right)^2}{2\eta_\sigma^2}\right]} \tag{39}$$

$$\phi(\delta_Q, 1) = \frac{1}{\sqrt{2\pi \eta_Q^2}} e^{\left[-\frac{\left(\delta_Q - \delta_Q^{mean}\right)^2}{2\eta_Q^2}\right]} \tag{40}$$

In numerical illustrations below we assume that the parameters take on the following values: the discount rate is $\rho = 5\%/yr$; the abatement cost exponent

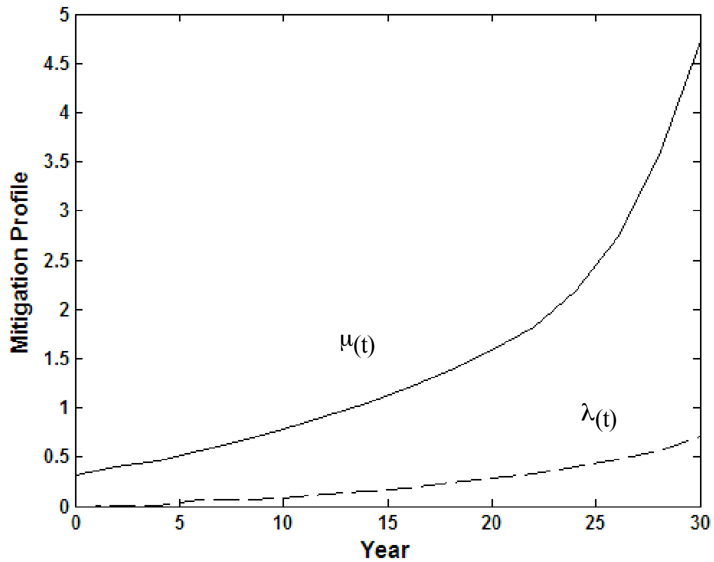

Figure 3: Optimal path for the control variable $\mu(t)$ and abatement $\lambda(t)$ for scenario in Figure 2. $\mu(t)$ can be larger than 1, whereas the abatement level $\lambda(t)$ is always less than 1. $\lambda(0) = 0$ and $\mu(0) = \frac{\dot{\lambda}(0)}{[\delta_\sigma + \delta_Q]}$

is $\nu = 2$; the declining carbon intensity of the economy is $\delta_\sigma = -1\%/yr$; and the long-term growth rate is $\delta_Q = 3\%/yr$. These values are to be seen as representative values rather as specific forecasts. We also vary the sum of uncertainties in technological change and economic growth, $\eta_\sigma^2 + \eta_Q^2$, from 0.5% to 1% and 2% per year to examine the impact of a range of values [2].

Figure 2 and Figure 3 show the emissions and abatement profiles for the reference non-stochastic case. Figure 4 and Figure 5 illustrates that for the stochastic case, the emissions paths are always lower, and the level of emissions reductions needed to meet a given target increase monotonically with increasing uncertainty the value of $\eta_\sigma^2 + \eta_Q^2$. Additionally the effect of the uncertainty in this value increases with a longe time horizon. This leads to the conclusion that the effect of stochasticity on optimal abatement strategies is to require a more aggressive approach to meet a given target.

The effect of stochasticity on total *amount* of mitigation required to meet the target is also is also strongly influenced by the time horizon for mitigation. Figure 6 shows that the amount of extra mitigation needed due to stochasticity

[2] Long term economic growth in much of the industrialized world has ranged close to 2% over the past 200 years, with a slightly faster average in the post World War II era of 2.4%. The coefficient of variation in growth over the post World War II period has been between 0.3 and 0.5 implying an inter-annual standard deviation of 0.7% to 1.2% (Dowrick & Nguyen, 1989). Since the 1950s decarbonization of the economy has proceeded in the US at an average rate of 1.6% per year, and an inter-annual variation that we calculated to be 0.5% (EIA, 2007). For further discussion of historical rates of decarbonization see Nakicenovic (1996).

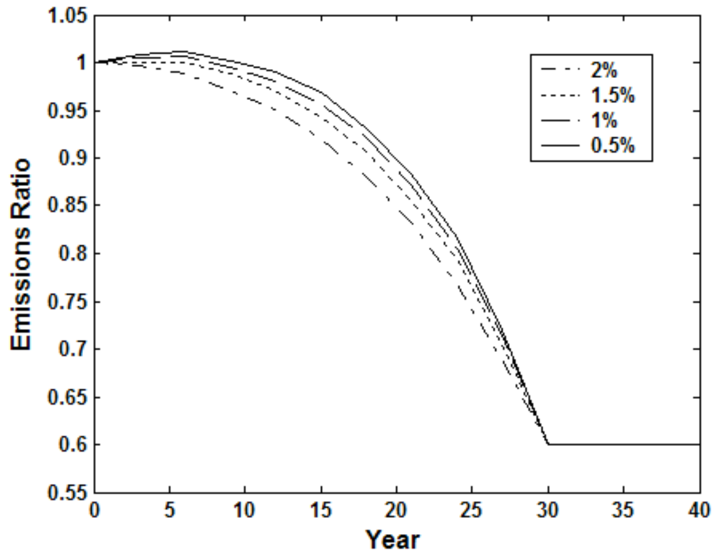

Figure 4: Optimal expected GHG emission profiles for stabilizing emissions after 30 years to 60% of those at $t = 0$. Values for stochasticity $(\eta_\sigma^2 + \eta_Q^2)$ set at 2%, 1.5%, 1% and 0.5%.

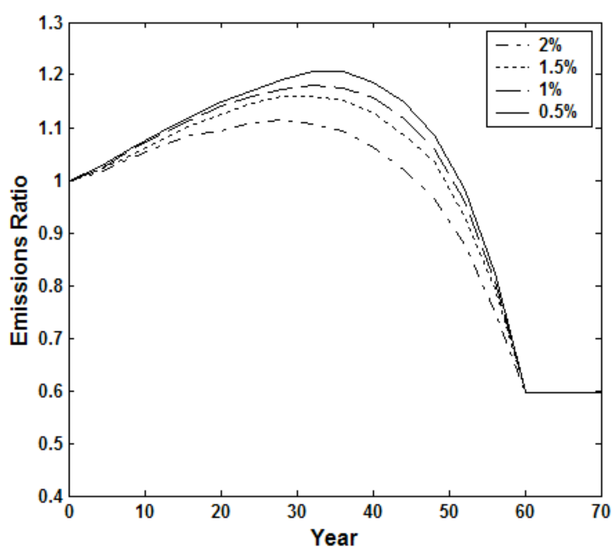

Figure 5: Optimal expected GHG emission profiles for stabilizing emissions after 60 years to 60% of those at $t = 0$. Values for stochasticity $(\eta_\sigma^2 + \eta_Q^2)$ set at 2%, 1.5%, 1% and 0.5%.

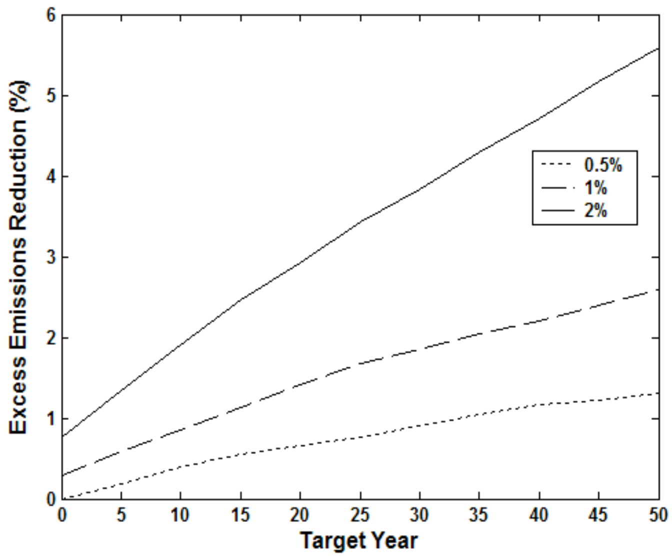

Figure 6: The influence of time horizon on the effect of stochasticity. This figure shows the total excess reductions in emissions (Integrated from $t = 0$ to $t = T$) required to meet a 40% reduction target as a function of time plotted for different values of stochasticity (0.5%, 1%, 2%). All other parameters fixed to those in Figure 4.

is greater as the time horizon, T, increases. In Figure 6 the total amount of excess reductions required to meet a 40% reduction target as a function of T relative to the case with perfect foresight are shown for values of stochasticity $(\eta_\sigma^2 + \eta_Q^2)$ equal 0.5%, 1% and 2% respectively. This translates to between 2–10 % of *total* emissions for the entire time period depending on the length of the time horizon.

The initial rate of abatement (at $t = 0$) $\dot{\lambda}(0)$ is quite sensitive to economic and technological uncertainties. The effect is difficult to elicit analytically but is illustrated numerically in Figure 7. From Figure 7, we see that the impact of stochasticity on the initial rate of abatement is strongest for high values of economic growth rates and low rates of exogenous decarbonization. For observed historical rates (cf. footnote 2), the initial level of abatement is between 30 to 70% higher.

In Figure 8 we show the influence of stochasticity on the cost of mitigation. The curve in the figure show the ratio of the expected mitigation cost given by $\big\langle Cost[\mu]\big\rangle = a\int_0^T \int \mu(t)^\nu \phi\big(\sigma(t)Q(t)\big) \cdot d\big(\sigma(t)Q(t)\big)e^{-\rho t} \cdot dt$ for two different scenarios with $T = 30$ years. The scenarios whose ratio is shown Figure 9 are: (1) the optimal scenario that includes stochasticity; (2) a scenario where a "sub-optimal" path and stochastic effects were ignored when in fact they were present, i.e., this trajectory would be optimal in absence of stochasticity. In

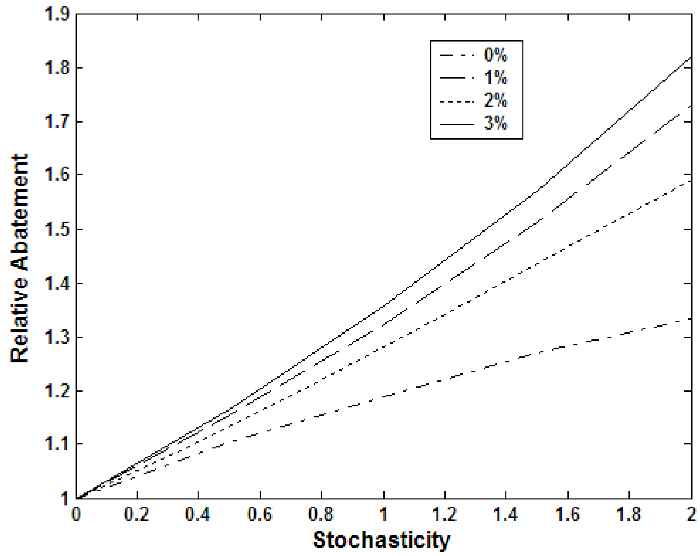

Figure 7: Effect of stochasticity on the relative value of the rate of abatament $\frac{\dot{\lambda}_{st}(0)}{\dot{\lambda}(0)}$ for different values *equationunclear* = 0%, 1%, 2% and 3% respectively, assuming 40% reduction in $T = 30$ years. All other parameters fixed to those in Figure 4

other words, the figure shows the price of ignoring stochasticity.

The message of Figure 8 is that the total mitigation cost increases if one ignores the uncertainties, and that these increases can range between 10–20%. As seen in Figure 9, the optimal path that incorporates stochasticity starts with higher levels of initial abatement i.e., the cost is initially higher. The sub-optimal path starts lower but eventually crosses over in order to meet the emissions target. The increase of cost towards the latter half of the period is such that the overall cost is higher. The zero-stochasticity sub-optimal path also implies that there is no adjustment around the optimal path to stochastic variations in economic and technological changes. In reality, such degree of precision in orchestrating emissions reductions is unlikely to be achieved. Figure 10 shows the probability density function for the ratio of the costs of the two paths. The optimal path that incorporates stochasticity is less expensive 70% of the time. This leaves only a 30% chance that the perfect foresight assumption will result in lowered costs.

8 Conclusions

We demonstrate in this paper is that it is possible to mathematically estimate the combined effect of the uncertainties in technological change and economic

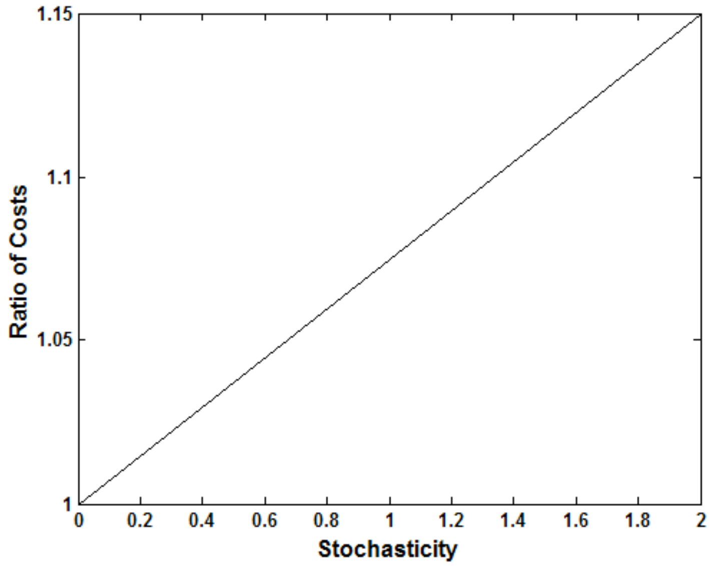

Figure 8: The effect of stochasticity on relative abatement costs. The dependent variable is the expected ratio of the cost of an optimal emissions path that ignores stochasticity versus one that does not. The calculation assumes that emissions will be reduced to 60% initial emissions ($T = 0$) at $T = 30$ years. All other parameters fixed to those given in Figure 4.

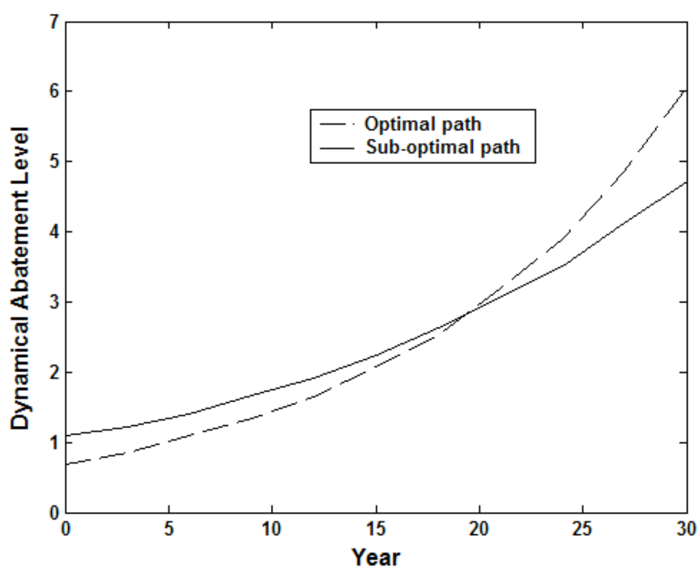

Figure 9: Comparison of the optimal path that accounts for stochasticity and a "sub-optimal" one that acts as though perfect foresight exists. All parameters fixed to those given in Figure 4

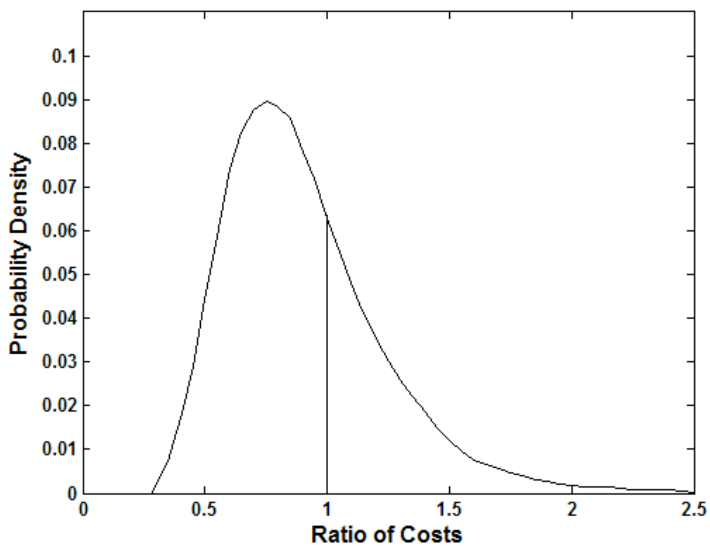

Figure 10: The probability density function for the ratio of total costs of the optimal and sub-optimal (perfect foresight assumption) abatement path shown in Figure 9. When the cost ratio is < 1, the optimal path is less expensive and vice-versa. The probability that the optimal path is less expensive than the sub-optimal path is the area to the left of cost-ratio $=1$ and is equal to 0.7, i.e., the optimal path is less expensive 70% of the time.

output on the optimal approach to greenhouse gas mitigation. The effect of these uncertainties, which manifest themselves as stochastic changes in growth rate and rates of decarbonization, is toward approaching mitigation more aggressively. Assuming historical levels of growth rates and rates of decarbonization (for a scenario requiring 40% reduction in 30 years) the uncertainty translates in an expected sense into: higher initial rates of abatement (30–70%); higher cumulative reductions (2–10%); lower costs of abatement (10–20%).

Why does the presence of uncertainties manifest itself in an optimal abatement path that requires greater and earlier abatement? Consider two categories of emissions paths. The first is a set of emissions paths (L), where the stochasticity acts to produce total emissions lower than those from the non-stochastic case. The second is the set of emissions paths (H), where the stochasticity acts to produce emissions that are higher than those from the non-stochastic case. The optimal abatement strategy has to be chosen such that on an *average* the emissions target $E_{GHG}(T)$is met at the least cost. For emissions paths, L, the stochasticity acts to produce an optimal path that requires lower abatement than the non-stochastic case, while for H, the opposite is true.

The cost function is a supra-linear function of the level of abatement ($\nu > 1$), with higher levels of abatement resulting in more than proportional costs. Emissions paths on H and those on L that have the same level of deviation from the non-stochastic case have very different cost differentials relative to the optimal expected path. As illustrated in Figure 10, there is an additional penalty in having to make corrections in path H when emissions are greater than those in the optimal abatement path. Thus, the presence of stochastic fluctuations requires abatement *earlier* to reduce the chances of being on a path in H. Earlier and more abatement serves as insurance to hedge against much higher costs later.

What if one were to act *as if* the stochasticity does not exist, when in fact, it does? When uncertainties are ignored, an optimal emissions path calls for lower levels of mitigation to meet the same target (cf Equation 13). In such a scenario, it becomes apparent at some point in time, that it is difficult to meet a target at current mitigation levels. As a consequence rapid and large amounts of abatement are now needed. The increase in abatement cost needed to meet a target if the effect of stochastic fluctuations is disregarded (Figure 9) are sizable and can grow rapidly with the size of the uncertainties.

Our model assumes that $\sigma(t)$ is independent of $\lambda(t)$. As a number of scholars have pointed out pressure of mitigation is expected to generate endogenous technological change, and $\sigma(t)$ can drop in response to increasing $\lambda(t)$ for a diversity of reasons: through a broad but purposive portfolio of R&D investments (Grubb et al., 2002); through technological learning in niche markets and subsequent diffusion to the rest of the economy (Grubler et al., 1999), aided by the development of performance standards to limit consumer choice (Azar & Dowlatabadi, 1999). Timing is critical to appropriate implementation of these insights and there are several reasons why greater early mitigation efforts may not lead to technical improvement. A stronger push for early mitigation might result in over investment in and lock-in of existing technologies thus 'crowding out' the development of new and better options; rent seeking behavior (e.g., of the kind evidenced in the recent corn ethanol "frenzy" in the US (Klein & LeRoy, 2007)) might lead to investment in mitigation actions that have negligible long-term impact on emissions. Clearly, there are many uncertainties in the pathways through which mitigation influences endogenous technological change. This analysis shows that outside the policy induced shifts in lock-in and patterns of R&D, uncertainty clearly points to the benefits of early deep mitigation. The relative magnitude of the policy induced technology effects vs. the autonomous effects need to be resolved in order to arrive at a generalized solution to this issue.

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