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# Optimal carbon emissions and the social cost of carbon over time under uncertainty

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**Abstract**

This paper uses the PAGE2002 model to find the path of $CO_2$ emissions that minimises the mean sum of the uncertain impacts of climate change plus the uncertain abatement costs, as well as the social cost of carbon under a variety of emission scenarios, including the optimal path. Compared to year 2000 emissions, optimal emissions are 34% higher in 2020, fall below year 2000 emissions in approximately 2050 and are 88% lower in 2100. Today's social cost of carbon is fairly insensitive to the exact scenario; its mean value is \$14 per tonne of carbon under the business as usual scenario, and \$12 under the optimal scenario, with 90% confidence intervals of about \$3–40 (all in \$US 1995). The mean SCC increases at about 3.2% per year in the optimal scenario.

**Keywords:** Optimal emissions, social cost of carbon, uncertainty.

## 1 Introduction

PAGE2002 is an integrated assessment model, with a time horizon of 2200, which has previously been used to make probabilistic estimates of the social cost of carbon under a variety of assumptions (Hope, 2006*a*,*b*; Wahba & Hope, 2006; Stern et al., 2006). One reason for calculating the probability distribution of the social cost of carbon is so that it can be compared to the probability distribution of the marginal abatement cost of $CO_2$, to see whether cutbacks in emissions can be economically justified.

This paper takes the next step and uses a genetic-algorithm-based optimiser with the PAGE2002 model to find explicitly the path of emissions that minimises the mean sum of the uncertain impacts plus the uncertain abatement costs. It also reports the social cost of carbon under a variety of emission scenarios, including the optimal path.

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**Table 1:** Abatement cost parameters in PAGE2002

| | min | **most likely** | max |
|---|---|---|---|
| Low cost abatement in EU ($/t) | -20 | **10** | 40 |
| Additional cost in EU ($/t) | 20 | **35** | 50 |
| Cost multiplier for other regions | 0.6 | **0.8** | 1 |
| Low cost range in Annex 1 (% of base year) | 30 | **50** | 100 |
| Low cost range in LDCs (% of base year) | 50 | **100** | 200 |

## 2 Inputs

The Common Poles Image (CPI) scenario is used as the business as usual scenario, and the GDP, population and non-$CO_2$ greenhouse gas emissions from this scenario are used throughout the analysis (den Elzen et al., 2003). However the impact and abatement cost assumptions are the default PAGE2002 values. The impact parameters are described in Hope (2006*a*), but the abatement cost parameters need to be described here.

In PAGE2002, the abatement cost depends on the percentage by which $CO_2$ emissions in each region fall below the business as usual scenario. Three uncertain parameters are used to represent abatement costs in each region. The first is the cost of the cheapest control measures in $ per tonne of $CO_2$ abated. The second is the maximum percentage of base year emissions that can be cut back by the cheap control measures. The third represents the additional cost in $ per tonne of $CO_2$ for reductions in excess of this. Cost parameters in the non-focus regions differ from the values for the focus region by a regional multiplier. The values taken by these parameters in this study are shown in Table 1. So, using the most likely values as an example, the abatement cost in the EU is taken to be $10 per tonne of $CO_2$ for cutbacks up to 50% of base year values, and $45 ( $= 10 + 35$) per tonne of $CO_2$ for cutbacks beyond this. All values are in 1995 $US, and are assumed not to change with time.

These abatement cost parameters have remained relatively unchanged in PAGE since the previous versions of the model, PAGE91 and PAGE95, and represent an attempt to span the range of estimates available in the literature, from the initially negative costs found by Barker et al. (1993), using recycled carbon taxes, to the higher values typically reported by top-down macro-economic models (Hope et al., 1993; Plambeck et al., 1997). The lower costs in other regions than the EU reflect the smaller remaining opportunities for low cost energy efficiency given the high energy prices already in place in the EU, and the possibility of lower cost construction and civil engineering works in the lower wage economies of the LDCs.

# 3 Results

All impacts and costs are measured in 1995 $US with market exchange rates, as stipulated by the Innovation Modelling Comparison Project (IMCP) (Grubb et al., 2006). A pure time preference rate with a triangular distribution with minimum value of 1% per year, most likely value of 2% per year, and maximum value of 3% per year, and an elasticity of utility with respect to consumption with a triangular distribution with minimum value of -1.5, most likely value of -1, and maximum value of -0.5 are used throughout the analysis.

Abatement costs are calculated for the 550, 500 and 450 ppm $CO_2$ emission paths from the IMCP, developed by Tom Wigley using the TAR version of the MAGICC model (Wigley, 2003). Due to a feedback loop in PAGE2002's carbon cycle that simulates the ocean's decreasing carbon sequestration ability as the temperature rises (Hope, 2006*a*), PAGE2002's mean expected concentrations in 2100 are higher than the stated value for all three of the scenarios used by around 70 ppm, with a fairly broad range. Therefore these paths are described as '550', rather than 550, in this paper.

The abatement costs are found both for equal percentage cutbacks from 2000 levels across all regions, and for the sharing of the specified cutbacks between Annex 1 and non-Annex 1 countries that produces the lowest mean total abatement costs.

The model is also run to find the emission paths in Annex 1 and non-Annex 1 countries that minimise the mean sum of impacts plus costs; this is called the 'optimal' scenario, rather than the optimal scenario, to recognise that it is only optimal if all the input assumptions are accepted and the impact calculations are complete. All optimisations are performed under uncertainty using the RiskOptimiser genetic algorithm program (Palisade, 2004).

The present day social cost of carbon (SCC) is reported for each of these five scenarios (CPI, '550', '500', '450' and 'optimal' emissions), and the evolution of the SCC over time is found for the 'optimal' scenario.

## 3.1 Emissions

Figure 1 shows global emissions of $CO_2$ in the five scenarios. The CPI business as usual (BAU) scenario sees global $CO_2$ emissions roughly double by 2060 before slowly declining thereafter. The '550' emission path rises slightly above the CPI scenario in 2020 and then declines. The '500' path rises to 2020, but not as far as the CPI scenario and then declines. The '450' path declines after 2010.

The 'optimal' path is close to the '500' scenario up to 2060, but then drops below the '450' scenario by 2080. Cutbacks are 15% of BAU emissions in 2020, 32% in 2040, 60% in 2060, 84% in 2080 and 93% in 2100. Compared to year 2000 emissions, 'optimal' emissions are 34% higher in 2020, and 28% higher in 2040; they fall below year 2000 emissions in approximately 2050 and are 19% lower in 2060, 69% lower in 2080 and 88% lower in 2100.

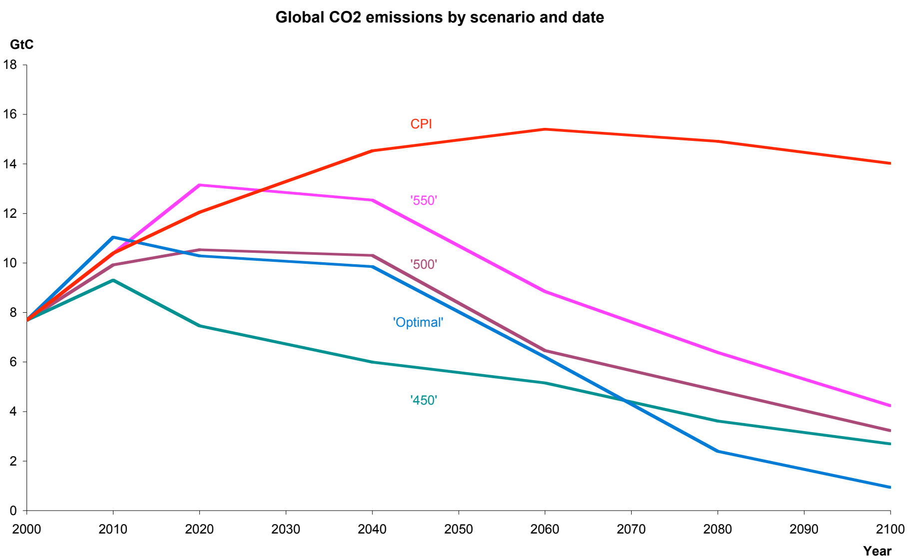



**Figure 1:** Global $CO_2$ emissions by scenario and date

The optimal shares for Annex 1 and other regions (LDCs) which minimise total mean abatement costs in the '550', '500', '450' and 'optimal' scenarios are shown in Figure 2.

In the '550', '500' and '450' scenarios, the optimal (least-cost) solution has emissions for the LDCs above the equal cutbacks path, and emissions for Annex 1 below the equal cutbacks path. In the 'optimal' scenario, LDC emissions reach zero by 2100; this is a consequence of the simple stepwise abatement cost curve assumed in the model, which has a constant marginal abatement cost once the more expensive portion of the curve is reached.

Figure 2 might give the impression that the major cutbacks are in the Annex 1 countries, but Figure 3 shows the cutbacks from the CPI scenario in each region and year as a percentage of the base year emissions in the region.

In nearly every scenario and date, the cutbacks from the CPI business as usual scenario are larger in the LDCs than in the Annex 1 regions. This is an unsurprising consequence of the assumptions in Table 1 that more cheap cutbacks are available in LDCs than in Annex 1 countries. The negative cutbacks to 2010 in the 'optimal' scenario are a consequence of the BAU scenario being specified with some uncertainty in PAGE2002, so the 'optimal' scenario stays close to the upper bound of this uncertainty initially to avoid incurring even a chance of abatement costs.

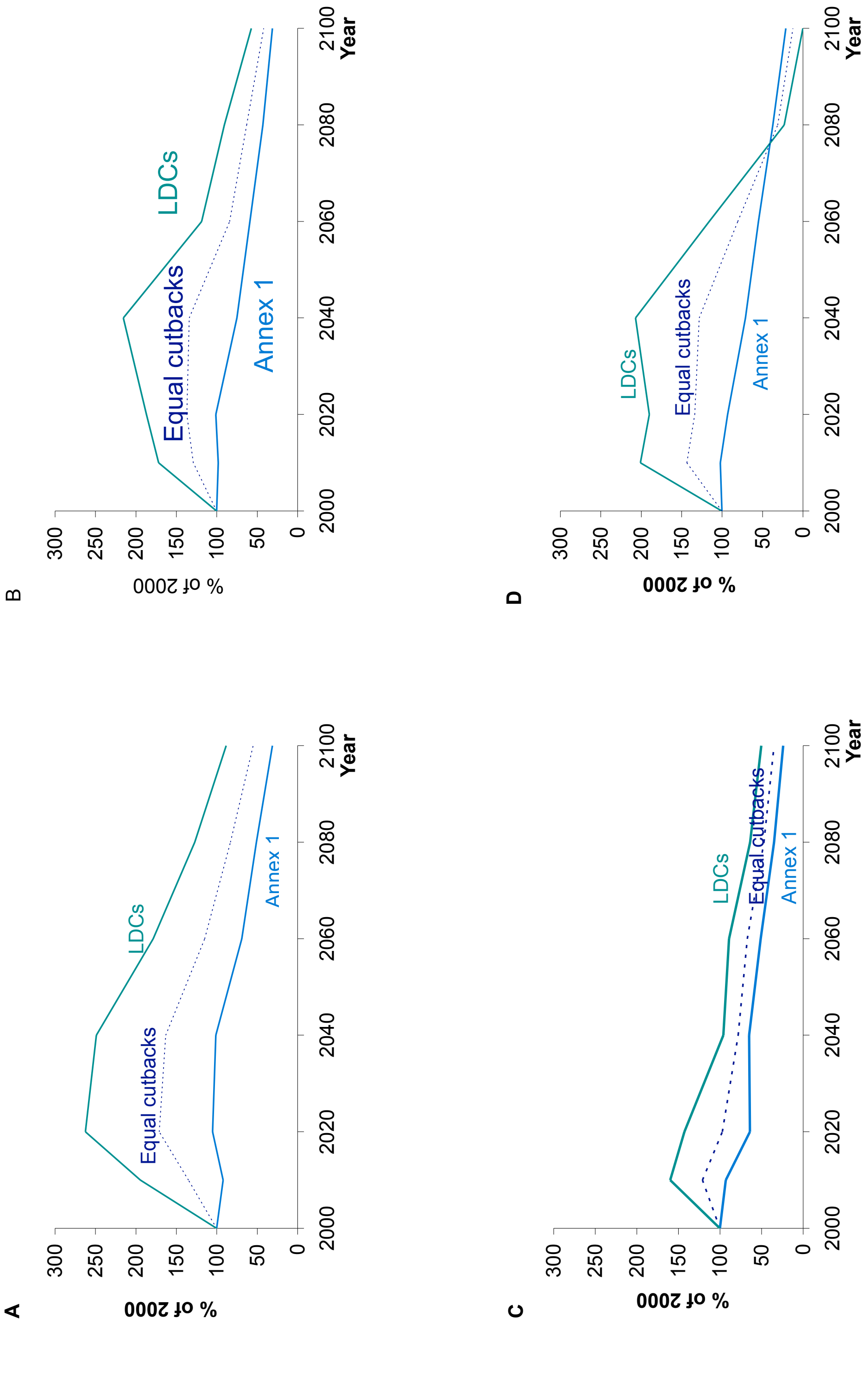



**Figure 2:** $CO_2$ emissions by region and date, optimal shares. A: '550' scenario; B: '500' scenario; C: '450' scenario; D: 'optimal' scenario.

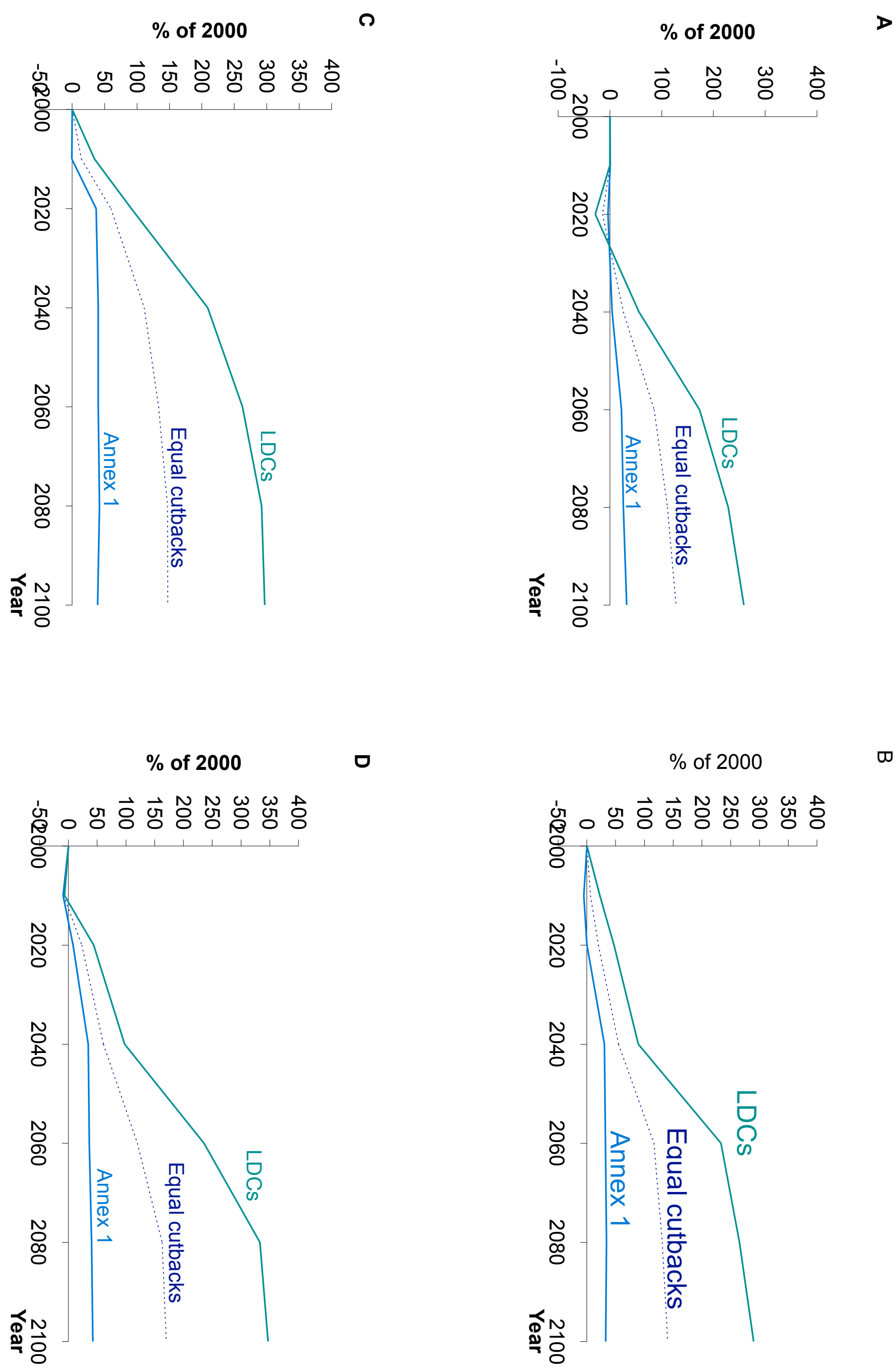



**Figure 3:** $CO_2$ cutbacks by region and date, optimal shares. A: '550' scenario; B: '500' scenario; C: '450' scenario; D: 'optimal' scenario.

**Table 2:** $CO_2$ concentrations in 2100 by scenario (ppm). Source: PAGE2002 model runs.

| | 5% | **mean** | 95% |
|---|---|---|---|
| CPI | 638 | **712** | 792 |
| '550 ppm' | 549 | **607** | 676 |
| '500 ppm' | 514 | **566** | 625 |
| '450 ppm' | 478 | **521** | 572 |
| 'optimal' | 495 | **542** | 597 |

## 3.2 Concentrations

The atmospheric $CO_2$ concentrations in 2100 for each of the scenarios are shown in Table 2

The mean concentrations for the '550', '500' and '450' ppm scenarios are higher than their nominal values by about 60 to 70 ppm. This is because PAGE2002 contains an estimate of the extra natural emissions of $CO_2$ that will occur as the temperature rises (the mechanism is actually mainly a decrease in absorption in the ocean (Intergovernmental Panel on Climate Change, 2001, p218), but the effect is the same as an increased emission). The mean excess concentration of 60 -70 ppm is in line with simulations reported by the IPCC (Intergovernmental Panel on Climate Change, 2001, p220). The 5%–95% range is about plus or minus 50 ppm.

## 3.3 Global temperature

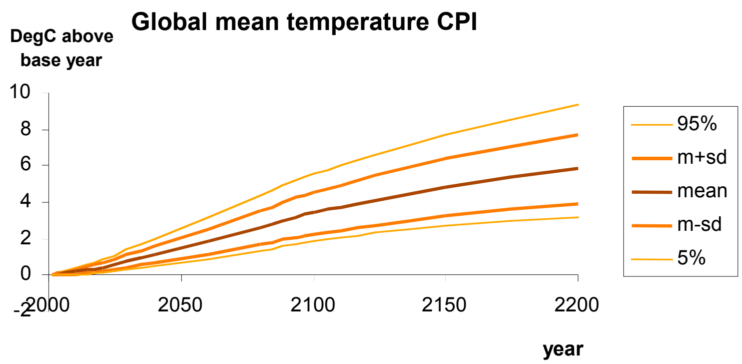
Figure 4 shows the global mean temperature rise since the base year, 2000, which has a global mean temperature 0.5°C above pre-industrial levels, for the CPI business as usual scenario, and the 'optimal' scenario. The increasing uncertainty with time is clearly visible. Results for the '550' scenario lie midway between the CPI and 'optimal' scenario; the '500' scenario is very similar to the 'optimal' scenario, and the '450' scenario is slightly below the 'optimal' scenario.

The rise in global mean temperature above pre-industrial levels in 2100 for each of the scenarios is shown in Table 3. The differences between the scenarios are not large in 2100; they diverge more in the 22$^{nd}$ century.

## 3.4 Impacts and costs

Figure 5 shows the mean total impacts and abatement costs from each of the scenarios. They are aggregated over the period 2000–2200 and discounted back to the base year 2000.

The costs are shown both with equal percentage cutbacks, and with the sharing of cutbacks that minimises total abatement costs. The impacts and a.

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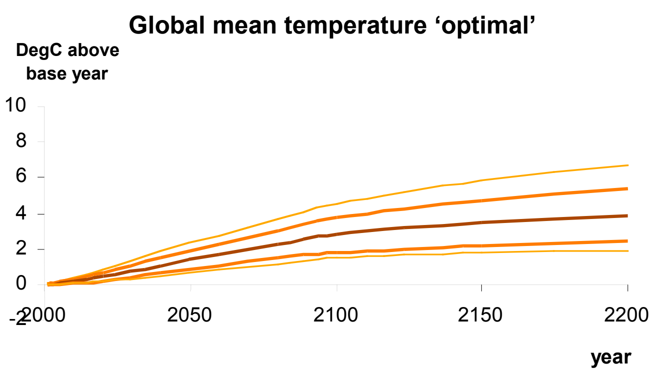


**Figure 4:** Global mean temperature by year. A: CPI scenario; B: optimal scenario

**Table 3:** Global mean temperature in 2100 by scenario (°C). Source: PAGE2002 model runs.

| | 5% | **mean** | 95% |
|---|---|---|---|
| CPI | 2.4 | **3.9** | 6.0 |
| '550 ppm' | 2.1 | **3.6** | 5.5 |
| '500 ppm' | 2.0 | **3.3** | 5.2 |
| '450 ppm' | 1.8 | **3.1** | 4.8 |
| 'optimal' | 2.0 | **3.3** | 5.1 |

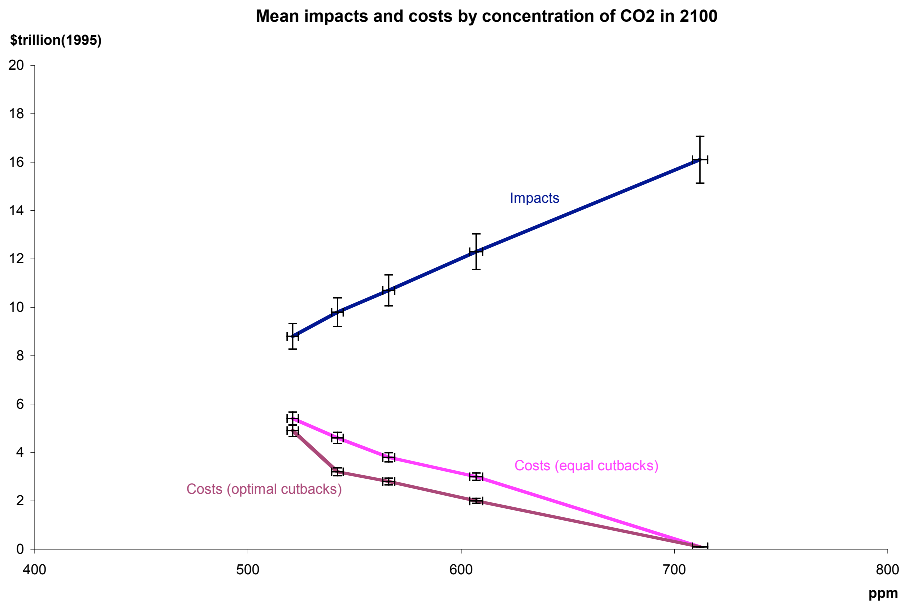


**Figure 5:** Mean impacts and costs by mean concentration of $CO_2$ in 2100.

costs are graphed against mean $CO_2$ concentrations in 2100. Reading from the left, the scenarios are '450', 'optimal', '500', '550' and CPI. The error bars show the 95% confidence intervals for the mean concentrations, impacts and costs, as the results are based on 1000 iterations of the PAGE2002 model.

For the '550' and '500' scenarios, the mean preventative costs are about $1.0 trillion lower with optimal sharing of cutbacks than with equal cutbacks. For the '450' scenario, costs are about $0.5 trillion lower.

Figure 6 shows the sum of mean impacts and preventative costs graphed against mean concentration of $CO_2$ in 2100. The costs for the '450', '500' and '550' scenarios are for the optimal sharing of emissions across regions. Reading from the left, the scenarios are '450', 'optimal', '500', '550' and CPI. The 'optimal' scenario does have the lowest mean total. The error bars are again the 95% confidence intervals.

Table 4 shows the total impacts of the scenarios. The mean value for the 'optimal' scenario is about 40% lower than for the CPI, even though its mean temperature rise in 2100 is only about 0.6°C lower. The difference between the scenarios increases in the $22^{nd}$ century, and this is when much of the contribution to total discounted impacts comes.

Table 5 shows the abatement costs of the scenarios. The costs of the CPI scenario are not exactly zero, because PAGE2002 includes uncertainty about exactly what the zero cost emission scenario is. The 5% point on the abatement cost distributions are negative for some scenarios, because there is a small chance

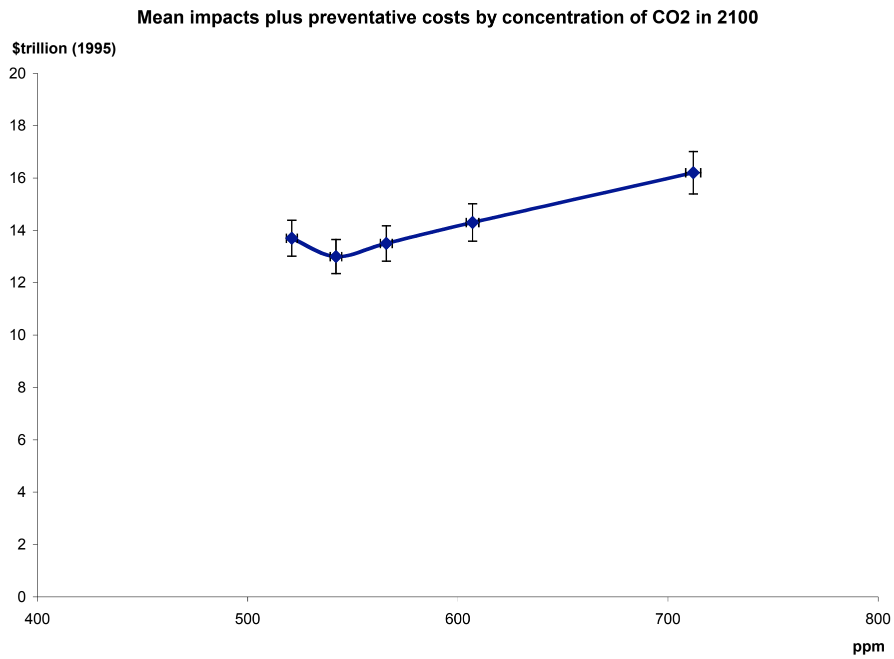


**Figure 6:** Mean impacts plus abatement costs by mean concentration of $CO_2$ in 2100.

**Table 4:** Impacts by scenario, 2000–2200 ($trillion (1995)). Source: PAGE2002 model runs

| | 5% | **mean** | 95% |
|---|---|---|---|
| CPI | 2 | **16.1** | 48 |
| '550 ppm' | 2 | **11.9** | 35 |
| '500 ppm' | 2 | **10.5** | 32 |
| '450 ppm' | 2 | **9.0** | 25 |
| 'optimal' | 2 | **9.8** | 28 |

**Table 5:** Abatement costs by scenario with optimal sharing of cutbacks, 2000–2200 ($trillion (1995)). Source: PAGE2002 model runs

| | 5% | **mean** | 95% |
|---|---|---|---|
| CPI | 0 | **0.1** | 0.3 |
| '550 ppm' | 0 | **2.0** | 5 |
| '500 ppm' | -1 | **2.8** | 7 |
| '450 ppm' | -1 | **4.9** | 12 |
| 'optimal' | -1 | **3.2** | 8 |

**Table 6:** Social cost of carbon by scenario, 2000 ($ per tonne C). Source: PAGE2002 model runs

| | 5% | **mean** | 95% |
|---|---|---|---|
| CPI | 3 | **14** | 41 |
| '550 ppm' | 3 | **13** | 37 |
| '500 ppm' | 2 | **13** | 43 |
| '450 ppm' | 2 | **13** | 37 |
| 'optimal' | 2 | **12** | 35 |

that the first tranche of cutbacks can be achieved while bringing benefits to the economy (through the recycling of tax revenues) as shown by the inputs in Table 1.

Table 6 shows the social cost of carbon in the scenarios. The insensitivity of the social cost to the exact scenario is clearly visible in this table.

Figure 7 shows the major influences on the social cost of carbon. The length of the line shows the strength of the influence (technically, the size of the partial rank correlation coefficient between the input and the SCC). The SCC is larger if the climate sensitivity is high, the ptp rate is low, and so on. Three points to note are that:

1. Four of the top eight influences are scientific; four are economic. This is a powerful argument for an integrated assessment.

2. The negative of the marginal utility of income enters the calculation in two ways: the higher it is, the higher the discount rate and the more the equity weights boost the impacts in poor regions. The chart shows that the SCC is larger, the smaller this parameter is. So its effect in increasing the discount rate outweighs its effect in increasing the impacts in poor regions.

3. The SCC is larger the smaller the tolerable temperature before there is a risk of a large-scale discontinuity. This is despite there being little chance of a discontinuity occurring before the 22nd century; it still has some effect on the SCC today.

The social cost of carbon varies with the date of emission. Figure 8 shows the SCC for the 'optimal' scenario up to 2060. Beyond that date, the time horizon of 2200 in PAGE2002 may make the estimates inaccurate. The increase in the mean SCC from 2000 to 2060 is about 3.2% per year.

## 4 Discussion

The 'optimal' scenario has substantial cutbacks of $CO_2$ emissions particularly in the latter half of the century; global emissions are about the same as base year emissions in 2050 but are only 12% of base year emissions in 2100. This is at variance with other results going back to the earliest work on this topic, such as Nordhaus (1993); Peck & Teisberg (1992) and Nordhaus & Boyer (2000) although not with Cline (1992). These earlier studies mainly gave smaller optimal cutbacks as they typically did not consider the effects of uncertainty or climate catastrophes, and had higher abatement costs without effective tax revenue recycling.

The mean value SCC of $14 per tonne of carbon under the CPI business as usual scenario is much lower than the mean value reported in the Stern review of $85 per tonne of $CO_2$ ($310 per tonne of carbon) for emissions that follow the SRES A2 business as usual scenario (Stern et al., 2006). This difference is entirely explained by the conversion from year 1995 $US with market exchange rates used here to year 2000 $US with purchasing power parity exchange rates in Stern, which multiplies the SCC values by about 3, and the use of a 0.1% per year pure time preference rate in Stern instead of a range of ⟨1,2,3⟩% per year used here, which multiplies the SCC values by about 7.

Since the 'optimal' scenario SCC values come from a scenario with an optimal set of emission cutbacks, they also represent the mean marginal abatement cost schedule under this scenario. If all the assumptions of the scenario are accepted, and all the impacts of climate change have been included, world permit prices or carbon taxes should also settle at these values. If the permit price for carbon emissions at any date is lower than the SCC at that date, this indicates too much $CO_2$ is being emitted, and vice versa.

Of course, it is by no means certain that all the scenario assumptions should be accepted; it has already been noted that the Stern review took a very different view on the rate of pure time preference. Also, many commentators argue for the use of purchasing power parity exchange rates, rather than market exchange rates (Schreyer & Koechlin, 2002), which increase the damages substantially as climate change impacts are felt largely in developing countries.

The PAGE2002 model does make a first attempt to include climate catastrophes in its damage calculations, but it omits some plausible socially-contingent

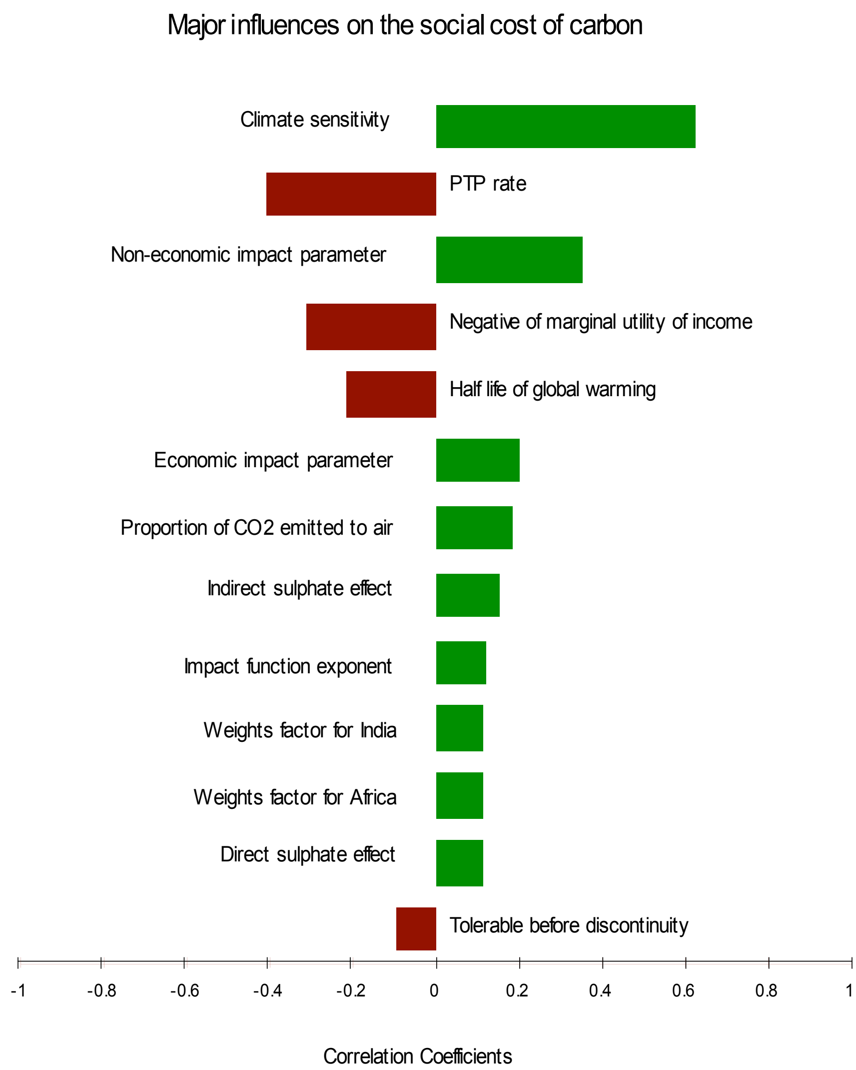


**Figure 7:** Major influences on the social cost of carbon

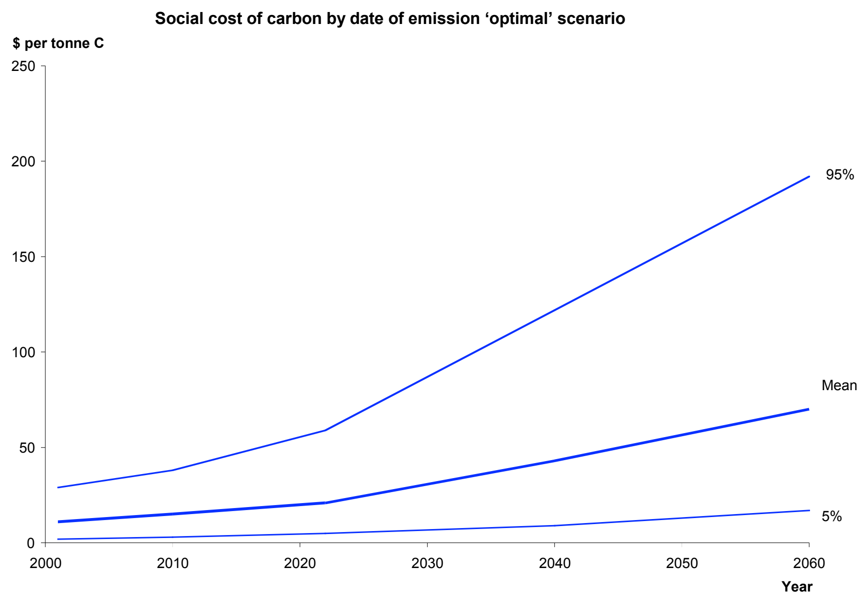


**Figure 8:** Social cost of carbon by date of emission, 'optimal' scenario

damages, such as the large number of climate refugees that might result from extreme climate change.

Conversely, the PAGE2002 abatement cost inputs may be lower than the current consensus. Their general pattern is not inconsistent with the findings of the IPCC fourth assessment report, which shows substantial global abatement possibilities in 2030 at a cost of less than \$20 (year 2000 \$) per tonne of $CO_2$, but the range for such low cost abatement possibilities is found there to be between 30 and 65% of year 2000 emissions, rather than the generally larger ranges shown in Table 1. However the IPCC range is only a 68% confidence interval, so values outside this range cannot be excluded (Intergovernmental Panel on Climate Change, 2007).

Further work to calculate optimal emissions, and the associated social cost of carbon over time, under these alternative sets of assumptions would certainly be worthwhile. One of the main purposes of this paper has been to demonstrate that such calculations under uncertainty are possible.

Finally, the optimal emissions in this paper implicitly assume that there is no opportunity for learning more about the major influences shown in Figure 7 before deciding which emission path to follow. If this is incorrect, and emission paths can be changed in response to better information about, say, the climate sensitivity, then the modelling approach described here can readily be extended to discover the monetary value of that better information.

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